Transforming Imprecise Probabilities into Partial Possibilities

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Abstract

This paper presents a procedure for transforming an imprecise probability assessment about an arbitrary finite set of events into a possibility assignment on the same set. To this purpose, a notion of partial possibility measure is introduced and characterized. These results represent a contribution to the more general research goal of defining an uncertainty interchange format and the relevant transformation procedures to be used in multi-agent systems.

Keywords: Partial possibility, Imprecise probabilities, Multi-agent systems, Uncertainty transformations.

1 Introduction

Defining transformation procedures between uncertainty representation formalisms is a key issue for enabling interoperability among heterogeneous uncertain reasoning systems, adopting different approaches to uncertainty. In particular, interoperability is a fundamental requirement in the development of multi-agent systems, namely systems composed by a set of autonomous software entities (called *agents*) interacting among them. A typical application scenario is a virtual marketplace, where independently owned software agents automate some of the activities related to the buying and selling of goods [8].

While consolidated proposals exist for inter-agent exchange of certain information (e.g. the Knowledge Interchange Format proposal of standard [7]), a relatively limited attention has been given in the literature to inter-agent exchange of uncertain information, in spite of its potential relevance in many applications. In [10] attention is paid in particular to the reuse and interoperability of existing expert systems, analyzing a family of transformations between the certainty factor model adopted in MYCIN and the subjective Bayesian model adopted in PROSPECTOR. We adopted a broader perspective in [1], where the problem of defining a sufficiently general uncertainty interchange format was considered. We identified coherent imprecise probability theory [12] as a suitable basis for the definition of an interchange format, due to its ability to encompass several well known theories (including precise probability, belief functions, and possibility) as special cases, and proposed transformation procedures from coherent imprecise probabilities into each of the three formalisms mentioned above. In [1] it was assumed that all agents share a common finite universe of discourse U and that the events about which they formulate their uncertainty judgements are taken from the powerset $\wp(U)$ of U. In other words, $U = \{e_1, \ldots, e_m\}$ is a finite partition made up of *atoms* (pairwise disjoint non-impossible events, whose logical sum is the certain event $\Omega = e_1 \vee \ldots \vee e_m$). Transformation procedures were proposed for the cases where the information exchange concerns either just one event or the whole $\wp(U) - \{\emptyset, \Omega\}$. However, a more general (and practical) case to be dealt with concerns the exchange of uncertainty quantifications about any set of events an agent considers interesting. This has, in particular, obvious advantages concerning the volume of information exchanged and the computational load involved by the transformation. Transformations between partial rather than complete uncertainty quantification assignments are therefore required. Coherent imprecise probabilities are defined on arbitrary sets of events (see section 3) and are therefore well-suited for handling also this case. Most well-known uncertainty theories, instead, require an uncertainty quantification assignment to be defined on an algebra of events. In order to enable partial information exchanges it is therefore necessary:

1. to provide a characterization of partial uncertainty assignments within each of the considered theories;
2. to define procedures for transforming an imprecise probability assignment into each kind of partial assignment in 1.

This paper aims at carrying out these steps for the case of possibility theory, which deserves a special attention for two reasons:

- as to our knowledge, a characterization of partial possibilities has not been provided yet in the literature (while, for instance, partial belief functions have been considered in [11]);
- as pointed out in [1], defining transformation procedures from imprecise probability into possibility is, in general, more critical than for other theories, due to the remarkable expressiveness gap the transformation has to cover and because some intuitively appealing requirements for these procedures turn out to be conflicting.

The paper is organized as follows. Section 2 provides a definition and a characterization of partial possibilities. After recalling the notion of coherent imprecise probability, section 3 introduces and discusses the criteria and assumptions underlying the procedure we propose and describe in detail in section 4. Finally, section 5 concludes the paper.

2 Partial possibilities

As well known (e.g. see [4]), a possibility measure Π on the powerset $\wp(U)$ of a finite partition $U = \{e_1, \ldots, e_m\}$ can be defined by assigning a possibility distribution $\pi(\cdot) : U \to [0, 1]$. In the following we will consider only *normal* distributions, i.e. distributions such that $\exists e_j \in U : \pi(e_j) = 1$. Then, $\forall E \in \wp(U)$, Π is given by:

$$\Pi(E) = \max_{e_i \Rightarrow E}\{\pi(e_i)\} \qquad (1)$$

with the assumption that $\Pi(\emptyset) = 0$.

Suppose now that a mapping $\Lambda : INTEV \to [0, 1]$ is given on the arbitrary finite set of events $INTEV = \{E_1, \ldots, E_n\}$ (to avoid trivial situations, we assume $E_i \neq \emptyset, E_i \neq \Omega, i = 1, \ldots, n$). The set $INTEV$ is assumed to include, in practical contexts, the events which actually are of interest for a software agent: in most cases, this set will not coincide with that of all non-trivial events of the powerset of any partition.

It would therefore be useful to define a notion of partial possibility to be applied to the partial uncertainty evaluation Λ on $INTEV$. Consider for this that the events of $INTEV$ may or may not be defined starting from an underlying partition U, but in any case the *partition* U_G *generated* by $E_1, \ldots, E_n$ can be obtained as the set of all logical products $E'_1 \wedge \ldots \wedge E'_n$, where each E'_i is alternatively replaced by either E_i or its complement E_i^c. Those logical products that are not impossible constitute the atoms of U_G. Partition U_G satisfies the following properties:

(i) any event $E_i \in INTEV$ is a logical sum of some atoms of the partition (those implying E_i);

(ii) U_G is the coarsest partition with the property (i).

We recall that a partition U is *coarser* than U' (or, equivalently, U' is *more refined* than U) iff every atom of U is a logical sum of atoms of U'. Clearly, property (i) is useful to relate partial to ordinary possibilities, as is done in the following definition.

Definition 1 $\Lambda : INTEV \rightarrow [0,1]$ *is a* partial possibility *(on $INTEV$) iff there exists a possibility measure $\Pi : \wp(U) \rightarrow [0,1]$, such that $\forall E_i \in INTEV, \Lambda(E_i) = \Pi(E_i)$, where $\wp(U)$ is the powerset of a finite partition U satisfying property (i) above.*

The next lemma ensures that the concept of partial possibility is well-defined, in the sense that it does not depend on the choice of U within the class of the finite partitions having the property (i). In particular, one may refer to U_G, i.e. the coarsest of these partitions.

Lemma 1 *If $\Lambda : INTEV \rightarrow [0,1]$ is not a partial possibility with respect to a possibility defined on $\wp(U_G)$, then it is not a partial possibility even referring to any possibility defined on $\wp(U)$, where U is a finite partition more refined than U_G.*

Proof. We equivalently show that whenever Λ is the restriction on $INTEV$ of a possibility measure Π on $\wp(U)$, then it is also the restriction of a possibility measure Π' on $\wp(U_G)$. In fact, let Λ be a partial possibility with respect to Π, and let π be the underlying possibility distribution defined on $U = \{\omega_1, \ldots, \omega_t\}$. Any event of $INTEV$ is a logical sum of atoms of $U_G = \{e_1, \ldots, e_m\}$: consider one of them, let it be $E_i = e_1 \vee \ldots \vee e_k, k < m$. Then

$$\Pi(E_i) = \max_{\omega_h \Rightarrow E_i} \{\pi(\omega_h)\} = \tag{2}$$
$$\max\left(\max_{\omega_h \Rightarrow e_1} \{\pi(\omega_h)\}, \ldots, \max_{\omega_h \Rightarrow e_k} \{\pi(\omega_h)\}\right)$$

Put $\pi'(e_i) = \max_{\omega_h \Rightarrow e_i}\{\pi(\omega_h)\}$ to assign possibility values to the atoms $e_1, \ldots, e_k$ of U_G. By considering all the events in $INTEV$, this assignment can be easily extended to other atoms to obtain a complete possibility distribution π' on U_G, and, applying (1), a possibility measure Π' on $\wp(U_G)$. Clearly, by construction we have $\Lambda(E_i) = \Pi'(E_i), \forall E_i \in INTEV$, i.e. Λ is a partial possibility with respect to U_G. ■

The next proposition gives a simple answer to the problem of establishing whether a given uncertainty assessment Λ on $INTEV$ is a partial possibility. For ease of notation, suppose that $1 \geq \Lambda(E_1) \geq \ldots \geq \Lambda(E_n) \geq 0$.

Further let $v_1 > \ldots > v_m, m \geq 1$, be the distinct values assumed by $\Lambda(\cdot)$, and define for $j = 1, \ldots, m$:

$$A_j^* = \bigvee (E_i : \Lambda(E_i) = v_j) \tag{3}$$
$$A_j = \bigvee_{k=j}^{m} A_k^* \tag{4}$$
$$A_{m+1} = \emptyset \tag{5}$$

Proposition 1 *An uncertainty assessment $\Lambda : INTEV \rightarrow [0,1]$ is a partial possibility iff both of the following conditions hold:*

$$\Lambda(E_1) = 1 \vee A_1 \neq \Omega \tag{6}$$
$$\forall j, \forall E_i : \Lambda(E_i) = v_j,$$
$$\exists e_k \in U_G : e_k \Rightarrow E_i \wedge A_{j+1}^c \tag{7}$$

Proof. Suppose first that both (6) and (7) hold. A possibility distribution function $\pi(\cdot)$ can be assigned on U_G as follows:

1. for $j = 1, \ldots, m$, for every $E_i : \Lambda(E_i) = v_j$ select one $e_k \in U_G : e_k \Rightarrow E_i \wedge A_{j+1}^c$ and put $\pi(e_k) = \Lambda(E_i)$;
2. if $\Lambda(E_1) < 1$, by (6) it is $A_1 = E_1 \vee \ldots \vee E_n \neq \Omega$, and we have $e_0 = E_1^c \wedge \ldots \wedge E_n^c \neq \emptyset, e_0 \in U_G$. Put then $\pi(e_0) = 1$;
3. assign $\pi(\cdot) = 0$ to all the remaining atoms of U_G to complete the definition of the possibility distribution π on U_G.

Then the possibility measure Π obtained from π applying (1) is such that $\Pi(E_i) = \Lambda(E_i), \forall E_i \in INTEV$.

Conversely, suppose now that either (6) or (7) do not hold.
If (6) is false, it is $\Lambda(E_1) < 1 \wedge A_1 = \Omega$. Hence every atom of U_G implies (at least) one $E_i \in INTEV$, and whatever possibility distribution π is assigned on U_G, this holds in particular for every e_j satisfying the normality condition $\pi(e_j) = 1$. Therefore, by (1) at least one $E_i \in INTEV$ should be given $\Pi(E_i) = 1$ thus yielding $\Pi(E_i) \neq \Lambda(E_i)$.
If (7) does not hold, this means that $\exists E_i, \Lambda(E_i) = v_j, j < m$ such that $E_i \wedge A_{j+1}^c = \emptyset$, i.e. $E_i \Rightarrow A_{j+1}$. Consider now any possibility measure Π on $\wp(U_G)$ such that $\Pi(E_i) =$

$\Lambda(E_i)$. By (1), we have that $\exists e_k \in U_G : e_k \Rightarrow E_i, \pi(e_k) = \Lambda(E_i)$. However, since $E_i \Rightarrow A_{j+1}$, we have that $\forall e_k \Rightarrow E_i, \exists E_s \in INTEV : e_k \Rightarrow E_s, \Lambda(E_s) < \Lambda(E_i)$. This entails that it is not possible to obtain a possibility measure Π with $\Pi(E_i) = \Lambda(E_i)$, without having some E_s such that $\Pi(E_s) \neq \Lambda(E_s)$. Therefore, Λ is not a partial possibility. ∎

Example 1. Let $U = \{e_1, \dots, e_9\}$, $INTEV = \{E_1, E_2, E_3, E_4\}$, where $E_1 = e_1 \vee e_2 \vee e_3$; $E_2 = e_3 \vee e_4 \vee e_5 \vee e_6$; $E_3 = e_5 \vee e_6 \vee e_7$; $E_4 = e_2 \vee e_3 \vee e_4 \vee e_5 \vee e_8$. Any assignment Λ on $INTEV$ such that $1 > \Lambda(E_1) > \Lambda(E_2) > \Lambda(E_3) > \Lambda(E_4) > 0$ can not be a partial possibility, because (7) does not hold for $j = 2$: $A_2^* = E_2$, $A_3 = E_3 \vee E_4$, every $e_k \in U$ implying E_2 implies also $E_3 \vee E_4$. As a matter of fact, because of (1) any partial possibility on $INTEV$ must assign to either E_3 or E_4 the same or a higher value than the one it assigns to E_2.

3 From imprecise probabilities to partial possibilities: criteria and constraints

We refer to the theory of coherent imprecise probabilities developed in [12]. It is assumed that the following conjugacy relation holds between lower ($\underline{P}$) and upper ($\overline{P}$) probabilities: $\underline{P}(E) = 1 - \overline{P}(E^c)$. We can therefore restrict our attention to lower or, as in our case, upper probabilities only.

Coherent upper probabilities are defined in [12] as follows: given an arbitrary (finite or not) set of events S, $\overline{P}(\cdot)$ is a *coherent upper probability* on S iff,

$\forall m, \forall E_0, \dots, E_m \in S, \forall s_i \geq 0, i = 0, \dots, m$, defining $I(E)$ as the indicator of E ($I(E) = 1$ if E is true, $I(E) = 0$ if E is false) and putting $\overline{G} = \sum_{i=1}^{m} s_i[\overline{P}(E_i) - I(E_i)] - s_0[\overline{P}(E_0) - I(E_0)]$,
it is true that $\max \overline{G} \geq 0$.

Coherent upper (and lower) probabilities have a clear behavioral interpretation in terms of betting schemes [12] and encompass several existing theories as special cases [13]. In particular, a possibility measure is a coherent upper probability.

As pointed out in [1], the problem of transforming an imprecise probability into a less expressive uncertainty formalism has no throughout accepted solution: every transformation mechanism involves some arbitrariness and is questionable in some respect. A similar remark was given in [6] for precise probability/possibility transformations. Here we focus on the transformation from an upper probability assignment $\overline{P}(\cdot)$ into a partial possibility $\Pi(\cdot)$, both defined on the same set of interesting events $INTEV = \{E_1, \dots, E_n\}$. Two main transformation criteria can be identified (see [1] for more general considerations):

- *consistency*: since possibility is an intrinsically less precise measure, the resulting $\Pi(\cdot)$ should not be more informative than the original $\overline{P}(\cdot)$;
- *similarity*: the resulting $\Pi(\cdot)$ should differ as little as possible from $\overline{P}(\cdot)$.

Operationally, the consistency criterion is applied imposing that $\forall E_i \in INTEV, \Pi(E_i) \geq \overline{P}(E_i)$. This is a straightforward extension of the consistency principle proposed in [3], [6]. As for the similarity criterion, two kinds of similarity can be considered:

- *ordinal similarity*: the credibility order induced on events by $\overline{P}(\cdot)$ should be preserved by $\Pi(\cdot)$;
- *quantitative similarity*: some distance between $\Pi(\cdot)$ and $\overline{P}(\cdot)$ should be minimized.

Ordinal similarity is related to the preference preservation principle [6], while quantitative similarity aims at minimizing the distortion introduced by the transformation. As discussed in [1], these criteria are inherently conflicting: the constraints due to preference preservation are often very strong and may significantly widen the imprecision gap between $\Pi(\cdot)$ and $\overline{P}(\cdot)$. For this reason, no transformation procedure appears to be *optimal* in an absolute sense, since it can be rated differently, depending on the relative importance one ascribes to conflicting criteria.

The procedure we describe in the next section ensures the consistency of transformation result and privileges ordinal rather than quantitative similarity.

A motivation for preferring ordinal similarity is given by the basically ordinal nature of possibility measures. As stated in [5]: "Possibility and necessity measures are set-functions that can provide simple ordinal representations of graded belief. Their particular character lies in their ordinal nature". Hence, it is reasonable to assume that a software agent adopting possibility theory as its uncertainty model is mainly interested in the credibility ordering associated with other agents' evaluations, and would possibly accept relatively loose approximations of the sender's numerical evaluation.

From an operational perspective, computationally simpler procedures should be preferred. In our case, this also means trying to operate directly on the events of $INTEV$, without enumerating and explicitly considering all the atoms of U_G. The proposed procedure takes also this aspect into account.

4 A transformation procedure based on ordinal similarity

4.1 Procedure definition

Given a coherent upper probability $\overline{P}(\cdot)$ on $INTEV = \{E_1, \ldots, E_n\}$, we define a procedure producing a partial possibility $\Pi(\cdot)$ such that:

$$\begin{aligned} &\forall E_i \in INTEV, \\ &\qquad \Pi(E_i) \geq \overline{P}(E_i); \qquad (8) \\ &\forall E_i, E_j \in INTEV, i \neq j, \\ &\quad \overline{P}(E_i) = \overline{P}(E_j) \Rightarrow \Pi(E_i) = \Pi(E_j); \quad (9) \\ &\quad \overline{P}(E_i) > \overline{P}(E_j) \Rightarrow \Pi(E_i) \geq \Pi(E_j). \quad (10) \end{aligned}$$

Condition (8) enforces the already mentioned consistency principle, while (9) and (10) impose a weak form of the preference preservation principle. Among the partial possibilities respecting the above conditions, it would be desirable to find one minimizing the additional imprecision, which, as far as (8) holds, can be measured by:

$$\sum_{E_i \in INTEV} (\Pi(E_i) - \overline{P}(E_i)). \qquad (11)$$

As in section 2, it is assumed that $\overline{P}(E_1) \geq \ldots \geq \overline{P}(E_n)$; further, A_j^* and A_j are defined by (3) and (4). Moreover all the implication and incompatibility relations among the events of $INTEV$ are assumed to be known, as well as whether $\vee(E_i : E_i \in INTEV) = \Omega$.

The procedure we propose is based on Proposition 1. It first checks whether conditions (6) and (7) are satisfied. In Particular, condition (7), involving atoms of the generated partition, can be checked exploiting only information about implication and incompatibility, by considering its negation, namely:

$$\exists j, \exists E_i : \overline{P}(E_i) = v_j, E_i \wedge A_{j+1}^c = \emptyset \qquad (12)$$

or equivalently

$$\exists j, \exists E_i : \overline{P}(E_i) = v_j, E_i \Rightarrow A_j^* \wedge A_{j+1}. \quad (13)$$

If $\overline{P}(\cdot)$ already satisfies both (6) and (7), the procedure terminates by putting $\Pi(E_i) = \overline{P}(E_i), \forall E_i \in INTEV$. Otherwise the procedure enforces (6) and/or (7). Condition (6) can easily be adjusted, if necessary, by putting $\Pi(E_i) = 1, \forall E_i : \overline{P}(E_i) = v_1$; this also complies with (9). The modified assignment is processed further if condition (7) is not satisfied, i.e. if the set $NOTSAT$ of events satisfying condition (13) is not empty. The procedure loops over the events of $NOTSAT$, considered in order of increasing upper probability (the choice for this ordering is motivated in the next subsection, comment to Step 9). Each loop iteration deals with one event $E_{CAND} \in NOTSAT$ and modifies some assignments in order to enforce (7) for E_{CAND}, while also taking into account the requirements (8) $\div$ (10). Then the procedure recomputes all A_j^*, A_j, and $NOTSAT$, referring to the modified assignment and selects the next event to be adjusted (if any). The procedure terminates when $NOTSAT$ is empty.

A pseudo-code specification of the complete procedure is the following:

Step 1. $\forall E_i \in INTEV$ `put` $\Pi(E_i) := \overline{P}(E_i)$;

Step 2. `if` $(\vee(E_i : E_i \in INTEV) = \Omega \wedge \Pi(E_1) \neq 1)$

`then` $\forall E_i : \Pi(E_i) = \Pi(E_1)$ `put` $\Pi(E_i) := 1$;

BEGIN MAIN LOOP

Step 3. Determine the values $v_1 > \ldots > v_m, m \geq 1$, of the current assignments $\Pi(E_i)$;

`for` $j = 1, \ldots, m$
  `put` $A_j^* := \bigvee(E_i : \Pi(E_i) = v_j)$;

`for` $j = 1, \ldots, m$
  `put` $A_j := \bigvee_{k=j}^{m} A_k^*$;

`put` $A_{m+1} := \emptyset$;

Step 4. Determine the set $NOTSAT = \{E_i$ such that $\exists j : \Pi(E_i) = v_j, E_i \Rightarrow A_j^* \wedge A_{j+1}\}$;

`if` $NOTSAT = \emptyset$ `then` **EXIT** `else`

Step 5. Select $E_{CAND} \in NOTSAT$: $\Pi(E_{CAND}) \leq \Pi(E_i), \forall E_i \in NOTSAT$;
Let h be such that
$\Pi(E_{CAND}) = v_h, E_{CAND} \Rightarrow A_h^* \wedge A_{h+1}$;

Step 6. Determine the set $IMPLIED_<(E_{CAND}) = \{I_1, \ldots, I_k\}, k \geq 1$, where each $I_j = \bigvee E_r$ is a minimal (with respect to implication) sum of events of $INTEV$ such that $\Pi(E_r) < \Pi(E_{CAND}), \forall E_r$ in the sum, and $E_{CAND} \Rightarrow I_j \Rightarrow A_{h+1}$;

Step 7. $\forall I_j \in IMPLIED_<(E_{CAND})$ select among the events E_r forming I_j an event $E_{MAXj} : \Pi(E_{MAXj}) \geq \Pi(E_r) \wedge \overline{P}(E_{MAXj}) \geq \overline{P}(E_r), \forall E_r$;

Step 8. `put` $SMAX := \bigcup_{j=1,\ldots,k}\{E_{MAXj}\}$;
`put` $\Pi_{BASE} := \min_j(\Pi(E_{MAXj}))$;
`put` $BASES := \{E_b \in SMAX : \Pi(E_b) = \Pi_{BASE}\}$;

Step 9. $\forall E_l \in INTEV : \Pi_{BASE} < \Pi(E_l) < \Pi(E_{CAND})$ `put` $\Pi(E_l) := \Pi(E_{CAND})$;

$\forall E_l \in INTEV : \Pi_{BASE} = \Pi(E_l)$
`if` $(\exists E_b \in BASES : \overline{P}(E_l) \geq \overline{P}(E_b))$
`then put` $\Pi(E_l) := \Pi(E_{CAND})$;

Step 10. `goto` Step 3.

END MAIN LOOP

4.2 Step by step explanation

Step 1 initializes the values of Π, step 2 checks condition (6) and enforces its satisfaction if needed. Then the main loop begins. Step 3 computes the events A_j^* and A_j used to check condition (7) in step 4: if $NOTSAT$ is empty, Π is a partial possibility and the procedure exits, otherwise steps $5 \div 10$ are executed.
Step 5 selects in $NOTSAT$ an event E_{CAND} with minimal value of Π. Step 6: since $\Pi(E_{CAND}) = v_h$ and E_{CAND} implies A_{h+1}, E_{CAND} must imply at least one sum $\vee E_r$ of events such that $\Pi(E_r) < \Pi(E_{CAND}), \forall E_r$. One such a sum is said to be *minimal* if any of its events E_r is necessary for the implication to hold. The set $IMPLIED_<(E_{CAND})$ is therefore not empty and includes all such minimal sums: it can be derived from the implication relations among events, assumed to be known. Step 7 selects from each minimal sum I_j an event E_{MAXj} with the highest current value of Π and also the highest original value of $\overline{P}$ (events with different initial values of $\overline{P}$ may have the same value of Π at a given iteration, due to adjustments carried out in previous iterations). Step 8 selects the set $BASES$ of events whose Π value is minimal among all E_{MAXj} chosen in step 7.
Step 9 then enforces the satisfaction of condition (7) for E_{CAND} by necessarily increasing the value associated to each E_{MAXj} from the current $\Pi(E_{MAXj})$ to $\Pi(E_{CAND})$. In this way, for each I_j at least an event (E_{MAXj}) is "subtracted" from A_{h+1}, thus also removing E_{CAND} from $NOTSAT$. In order to satisfy conditions (9) and (10) we also have to increase the value of Π for all E_l whose Π value is between any $\Pi(E_{MAXj})$ (included) and $\Pi(E_{CAND})$, i.e. those events that would be unduly surpassed by some E_{MAXj} in the ordering. The required modifications are obtained by increasing all the Π values strictly included between Π_{BASE} and $\Pi(E_{CAND})$ for any $E_l \in INTEV$.
The events E_l such that $\Pi_{BASE} = \Pi(E_l)$ require an ad hoc treatment. The if clause referring to the comparison of the initial upper probability values is necessarily true for each E_l in the first loop iteration. However, it may be not so in subsequent iterations, when it may be the case that two events currently featuring the same value of Π had originally different values of $\overline{P}$. For this reason, the if clause checks that the adjustments for the values of Π are actually required by ordinal similarity with respect to the initial assignment $\overline{P}$. In this way "false equalities", i.e. equalities of the values of Π that do not correspond to equalities of the values of $\overline{P}$, do not give rise to unnecessary additional imprecision.

The procedure then goes back to step 3.

It has to be noted that Step 9 is based on the following consideration: since E_{CAND} has the lowest Π value in $NOTSAT$, $\forall E_l : \Pi(E_l) < \Pi(E_{CAND})$, it ensues that $E_l \notin NOTSAT$. Therefore, all such E_l satisfy condition (7), i.e. there is at least an atom $e_k \in U_G$ which implies E_l without implying any event E_i with $\Pi(E_i) < \Pi(E_l)$. This enables us to increase $\Pi(E_l)$ by increasing an underlying value $\pi(e_k)$ without affecting any event E_i with a lower value of Π. Note that operationally this can be made without explicitly identifying e_k and therefore without deriving U_G, since it suffices to know that such an e_k exists.

Example 2. To make a simple application of the procedure, we transform the assignment Λ in Example 1 into a partial possibility. In step 1, we put $\Pi(E_i) := \Lambda(E_i)$, the if clause in step 2 is not operating, step 3 determines $A_j^* = E_j, j = 1, \ldots, 4$, and all A_j, including $A_3 = E_3 \vee E_4$. As $A_2^* \wedge A_3 = E_2$, $NOTSAT = \{E_2\}$ is found at step 4. $E_{CAND} = E_2$, $I_1 = E_3 \vee E_4$, $E_{MAX1} = E_3$, $SMAX = BASES = \{E_3\}$, $\Pi_{BASE} = \Pi(E_3)$ are determined in steps 5÷8. At step 9, no event E_l satisfies the strict inequalities in the first line, so the corresponding command is ineffective, whilst the if clause is satisfied for $E_l = E_b = E_3$, giving $\Pi(E_3) := \Pi(E_2)$. While recomputing $A_j^*, A_j, j = 1, 2, 3$, the second iteration gives now $A_2^* = E_2 \vee E_3$, $A_3 = E_4$, and consequently $NOTSAT = \emptyset$. The current Π is a partial possibility.

4.3 Procedure properties

In this section we briefly discuss the termination, event selection ordering, and additional imprecision properties of the procedure.

The procedure terminates when $NOTSAT = \emptyset$, i.e. when the current assignment Π satisfies condition (7). When it does so, Π satisfies also (6), by step 2, and is therefore a partial possibility on $INTEV$ by Proposition 1.
To see that termination is guaranteed, note that at each iteration of the main loop at least one event of $INTEV$ is assigned a higher Π value, which is chosen among the numbers $v_1, \ldots, v_{m-1}$, whilst no Π assignment is ever lowered. This is operated by step 9, from which it appears that Π is raised at least for the event(s) in $BASES$. It follows that one iteration of the main loop modifies the events A_j^* by moving at least one event of $INTEV$ from the event, say A_s^*, it implied at the beginning of the main loop to an event, say A_r^*, recomputed at step 3 in the next iteration and formed by events of $INTEV$ having a common, higher assignment of Π. Subsequent iterations will progressively tend to empty those A_j^* formed by events with lower assignments of Π, possibly reducing the number m of different A_j^*. Unless it stops before, the procedure will, by necessity, progressively decrease m: in the worst case it will arrive at $m = 1$, $A_1^* = \vee(E_i : E_i \in INTEV)$. At this point the procedure terminates, because $NOTSAT$ is necessarily empty.

As for event selection, the main loop iterations examine the events E_{CAND} in increasing order of Π: this ordering is necessary to guarantee that the procedure is well-founded without requiring to explicitly deal with the generated partition U_G, as explained in the comment to step 9 above. The procedure does not specify how to select E_{CAND} when some events form a *tie*, i.e. have the same minimal value of Π within $NOTSAT$. It can be shown however that the selection order for tied events has no influence on the final result. In particular, any of these selection orders has the same effect as choosing as first event within the tie one which gives rise to the minimal value of Π_{BASE}. Such a selection would remove all the events in the tie from $NOTSAT$ in a single iteration.

Given the above mentioned ordering, at each iteration an adjustment for each I_j identified in step 6 is necessary to obtain a partial possibility. Any such adjustment adds imprecision, since there is an increase of Π with respect to $\overline{P}$ for at least an event. The procedure attempts to minimize such an addition, since it selects for each I_j the best choice, i.e. the event E_{MAXj} with the highest previous assignment. Among these locally best choices, the minimal value Π_{BASE} (i.e. the worst from the viewpoint of imprecision addition) is used in step 9, but this is imposed by (9) and (10).

5 Conclusions

In this paper we have introduced and characterized a notion of partial possibility measure assigned on a generic finite set of events and proposed a procedure for transforming a coherent upper probability into a partial possibility. The procedure is based on the consistency and ordinal similarity criteria and features the computationally advantageous property of not requiring the explicit derivation of a complete generated partition. Moreover it tends to minimize the additional imprecision introduced during the transformation, as far as this is allowed by the above main criteria. This work generalizes the framework of [1], that was limited to consider all events obtained from a given partition, and is related to previous results concerning precise probability/possibility transformations [6] and interval probability/possibility transformations [2]. In particular, by operating directly on atomic events, [2] deals with complete possibility distributions only. Future work includes the study and comparison of alternative transformation procedures based on different criteria, such as quantitative similarity and uncertainty preservation [9]. To employ the latter criterion, a function measuring the uncertainty of any imprecise probability assessment should be introduced. An interesting open problem is whether this can be done following the way discussed in [9] for the case of belief functions.

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